University of California, Berkeley
Physics 110B Fall 2004 (*Strovink*)

EXAMINATION 1

Directions: Do all three problems, which have unequal weight. This is a closed-book closed-note exam except for Griffiths, Pedrotti, a copy of anything posted on the course web site, and anything in your own handwriting (not a Xerox of someone else's writing). Calculators are not needed, but you may use one if you wish. Laptops and palmtops should be turned off. Use a bluebook. Do not use scratch paper – otherwise you risk losing part credit. Show all your work. Cross out rather than erase any work that you wish the grader to ignore. Justify what you do. Express your answer in terms of the quantities specified in the problem. Box or circle your answer.

Problem 1. (30 points)
Define

$$g^{\mu}{}_{\nu} \equiv \sum_{\sigma=0}^{3} g^{\mu\sigma} g_{\sigma\nu} \ ,$$

where $g^{\mu\sigma}$ and $g_{\sigma\nu}$ are elements of the same metric tensor

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

of flat spacetime. Denote by $T^{\nu\rho}$ an element of the arbitrary four-tensor

$$\begin{pmatrix} a & b & c & d \\ e & f & g & h \\ i & j & k & l \\ m & n & o & p \end{pmatrix}$$

(the first superscript of T is the row index, the second is the column index).

Calculate all 16 elements $U^{\mu\rho}$ of four-tensor U, where

$$U^{\mu\rho} \equiv \sum_{\nu=0}^{3} g^{\mu}{}_{\nu} T^{\nu\rho} \ .$$

Give your answer in the form of a 4×4 matrix whose elements are expressed in terms of $a, b, c, \ldots, n, o, p$.

Problem 2. (30 points)
A photon (massless, total energy E) that travels in the x direction collides with a proton (rest mass m) that initially is at rest. The photon is scattered elastically by the proton – no rest mass is created and none is destroyed. The photon emerges from the collision traveling in the y direction, perpendicular to its initial direction of motion. The proton's recoil direction and recoil energy are unmeasured. In terms of E and m, what is the total energy ϵ of the scattered photon?

Problem 3. (40 points)

(**a**) (20 points)
Lorentz frame $\mathcal{S}'$ moves in the x direction with rapidity η_0 relative to the lab frame $\mathcal{S}$, where "rapidity" refers to the fundamental ("boost") parameter of the Lorentz transformation. Lorentz frame $\mathcal{S}''$ moves in the x direction with rapidity η_0 relative to frame $\mathcal{S}'$. With what *speed* u does frame $\mathcal{S}''$ move with respect to the lab frame $\mathcal{S}$?

(**b**) (20 points)
Consider the situation described in part (**a**). In the lab frame $\mathcal{S}$ there exists a uniform fixed scalar potential V_0 and a nonuniform vector potential

$$\vec{A} = -\alpha y \, \hat{x} \ ,$$

where α is a constant. Calculate the electric field $\vec{E}''$ that is seen by an observer who is at rest with respect to Lorentz frame $\mathcal{S}''$. In case you didn't get part (**a**) exactly right, please leave your answer in terms of u.